Evaluation of Polyharmonic Vibrations Effects upon Human Body aiming Improvement of Norms concerning Limits and Time Exposure

Aurelia MIHALCEA
Research Institute for Construction Equipment and Technology – ICECON, 266 Pantelimon, 021652 Bucharest, Romania, cinetic@icecon.ro

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This paper deals with the behaviour of nonlinear elastic systems whose dynamic response is characterized by polyharmonic vibrations while subjected to given (kinematic) harmonic excitations. Under these conditions, beginning with the stage of equipment conception, the numerical evaluation for the system response is necessary in order to determine the sub harmonic and over harmonic components related to the excitation frequency. Basing on this, the level of the vibrations transmitted to human body could be assessed allowing adoption of the appropriate protection measures. Also, this approach can be used in case the linear elastic system becomes nonlinear during operation. This results in changing of the response spectrum with negative impact upon human health and safety. The present paper contains actual cases of spectrum changing for the vibrations transmitted while operating the vibrating construction machinery where the additional sub harmonic or over harmonic components exceed the rated level and affect the human body exposed to vibrations during the working process.

1. INTRODUCTION

The assessment of vibration effects transmitted to human body during the working process as a result of the contact with machine supporting points or surfaces is performed basing on specific norms. In present, the effects evaluation for the vibrations transmitted to human body is carried out basing on diagrams expressing the correlation between one kinematic parameter, usually acceleration (rms), vibration frequency and exposure time. The representation of the acceleration variation in respect to frequency for a given signal is based on the response of the human body-machine dynamic system under defined conditions concerning the interaction and physical behaviour as follows:

- the excitation has a stationary spectral composition under the rated regime;
- the human body-machine system is characterized by linear physico-mechanical connections (elastic and dissipating forces).

In practice, one could meet frequent situations when due to the excitation or the nonlinear behaviour of the human body-machine system, beside the fundamental response in frequency corresponding to $j\omega$ values within the measured signal, some sub harmonic signals $i\omega$, with $2i = 2j - 1$, where j = 1, 2, 3, …, n appear.

Thus, the response signal frequency spectrum is richer and the sub harmonic and over harmonic vibration acceleration values could exceed the rated curves for the fundamental response. This leads to malfunctions and erroneous or non-uniform assessment for the transmitted vibrations in case the polyharmonic character of the human body-machine system response as a result of causes that has generated it, is not taken into account.

Mainly, two major causes generating a polyharmonic signal with sub harmonic and over harmonic spectral components exist as follows:

- time-modification of the exciting factor for the human body-machine system due to some operating causes, additional perturbing factors;
- physico-mechanical structure modification of the human body-machine system involving significant physical (elastic and dissipating) nonlinearities able to change the spectral response as compared with those specific to the linear behaviour.

2. DYNAMIC RESPONSE OF THE HUMAN BODY—MACHINE SYSTEM

The human body-machine system could be considered as one freedom degree model having m - inertia, c - dissipating and k - elastic equivalent factors.

For the physical system having linear behaviour the parameters are constant (m = const, c = const, k = const) and in case the system has an elastic and dissipating nonlinear behaviour the corresponding parameters are functions of displacement, speed respectively, so that $c = c(\dot{x})$ and $k = k(x)$.

For the linear physical system subjected to harmonic excitation we have:

$$m\ddot{q} + c\dot{q} + kq = h(t) \tag{1}$$

where

- m is the constant inertia equivalent factor expressing the mass in translation motion with coordinate $q = x$ or J the mass inertia moment in the rotating motion with coordinate φ;
- c is the constant damping equivalent factor assuring the proportionality between the viscous force and the vibration speed;
- k represents the elasticity equivalent factor;

$h(t)$ is the exciting factor that could be a force $\mathrm{h}(t) = \mathrm{Q}(t)$ or acceleration $\mathrm{h}(t) = \frac{1}{m}\mathrm{Q}(t)$.

The exciting factor $h(t)$ can be expressed by a harmonic function as follows:

$$\mathrm{h}(t) = \mathrm{Q}(t) = H_0 \sin \omega t \tag{2}$$

as a harmonic exciting force, or

$$\mathrm{h}(t) = \frac{1}{m} H_0 \sin \omega t = \dot{q}_0 \sin \omega t \tag{3}$$

as a harmonic exciting acceleration.

Besides this, the exciting factor could be expressed by a polyharmonic force represented under the form of a Fourier series, having n-harmonic components spectral composition. Thus, we have:

$$\mathrm{h}(t) = Q_0 + \sum_{j=1}^{\infty} Q_n \sin(j\omega_1 t + \theta_n) \tag{4}$$

where

Q_0 is the constant force under steady regime for $\omega = 0$;

$$Q_n = \left(u_j^2 + v_j^2\right)^{1/2}$$

with

$$u_j = \frac{2}{T}\int_0^T \mathrm{Q}(t) \cos j\omega_1 t dt \ \text{ for } \ \mathrm{j} = 1, 2, \ldots, \mathrm{n}$$

$$\theta_j = \operatorname{arctg} \frac{a_j}{b_j}$$

with

$$v_n = \frac{2}{T}\int_0^T \mathrm{Q}(t) \sin j\omega_1 t dt \ \text{ for } \ \mathrm{j} = 0, 1, 2, \ldots$$

$$Q_0 = \frac{2}{T}\int_0^T \mathrm{Q}(t) dt$$

In case the polyharmonic excitation is of the form of an acceleration, we have:

$$\mathrm{h}(t) = a_0 + \sum_{j=1}^{\infty} a_j \sin(j\omega_1 t + \alpha_n) \tag{5}$$

where

- $a_0 = \frac{1}{m} Q_0$ is the initial acceleration corresponding to force Q_0 applied to the mass m;
- $a_n = \frac{1}{m} Q_j$ is the acceleration corresponding to the n-order harmonic.

For the nonlinear physical system externally excited we have:

$$m\ddot{q} + c(\dot{q})\dot{q} + k(q)q = h(t) \tag{6}$$

where the terms $c(\dot{q})$ and $k(q)$ introduce the nonlinearities appeared inside the physical system that initially does not have them. By developing as a power series $c(\dot{q})$ and $k(q)$ coefficients and in the first approximation adopting the solution under the form $q = A_1 \sin \omega_1 t$, equation (6) can be linearized so that the right member will contain the terms of the Fourier development. In this case the solution of equation (6) will have in instantaneous displacements the form:

$$q = A_1 \sin(\omega t + \varphi_1) + A_2 \sin(2\omega t + \varphi_2) + \ldots$$

or

$$q = \sum A_j \sin(j\omega t + \varphi_j) \tag{7}$$

and under the acceleration form we have:

$$a = \sum a_j \sin(j\omega t + \varphi_j) \tag{8}$$

where

$$a_j = -\omega_j^2 A_j \ \text{with} \ \mathrm{j} = 0, 1, 2, \ldots, \mathrm{n}$$

The system dynamic response given by relation (7) or (8) could be the result of a polyharmonic excitation when the physical system is linear as well as of a harmonic excitation when the system is nonlinear.

There are situations when due to the elasticity variation law $k = k(q)$ as a symmetric or asymmetric function in respect to displacement (linear or angular) q in the basic response of the excited system sub harmonic vibrations having frequency $\omega_i = i\omega = \frac{1}{2}(2j - 1)\omega$ as well as over harmonic vibrations having the frequency

$$\omega_k = k\omega = \frac{1}{2}(2j+1)\omega \quad \text{with} \quad j = 1, 2, \ldots, n$$

occur.

The spectral response in acceleration is represented by the basic signal

$$a_L = \sum_{j=1}^{n} a_j \sin(j\omega t + \varphi_j) \quad \text{with} \quad j = 1, 2, \ldots, n$$

for the linear physical system (L) completed by the signal induced by the i-order sub harmonics of the form

$$a_{NL}^{i} = \sum a_i \sin(i\omega t + \varphi_i)$$

with

$$i = \frac{1}{2}(2j-1) \text{ and } j = 1, 2, \ldots, n$$

for a nonlinear sub harmonic system (NL) and by the signal

$$a_{NL}^{k} = \sum a_n \sin(k\omega t + \varphi_k)$$

with

$$k = \frac{1}{2}(2j+1) \text{ and } j = 1, 2, \ldots, n$$

Thus we have the global response of the basic signal “enriched” with sub harmonic and over harmonic signal under the form:

$$a = \sum a_i \sin(i\omega t + \varphi_i) + \sum a_j \sin(j\omega t + \varphi_j) + \sum a_k \sin(k\omega t + \varphi_k) \qquad (9)$$

where *i*, *j*, *k* are given in Table 1.

Table 1. i, j, k values

j	1	2	3	4	5
$i\omega$	$\frac{1}{2}\omega$	$\frac{3}{2}\omega$	$\frac{5}{2}\omega$	$\frac{7}{2}\omega$	$\frac{9}{2}\omega$
$j\omega$	ω	2ω	3ω	4ω	5ω
$k\omega$	$\frac{3}{2}\omega$	$\frac{5}{2}\omega$	$\frac{7}{2}\omega$	$\frac{9}{2}\omega$	$\frac{11}{2}\omega$

3. COMPARISON BETWEEN THE POLYHARMONIC VIBRATIONS AND THE PERMISSIBLE VALUES

The system dynamic response in case of a complex polyharmonic system is compared with the admissible acceleration (rms) diagrams represented in frequency bands centred on spectral analysis frequency values in octave or 1/3 octave, according to SR ISO 2631. Thus, figure 1 illustrates the limit curves as positive slope lines corresponding to limit exposure to vibrations transmitted to human body within the relation human body-machine.

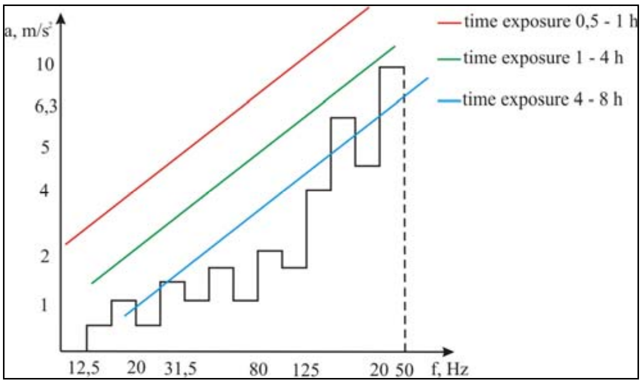

Figure 1. Limit curves for human exposure

The actual acceleration spectrum determined basing on instrumental way, represented in one third octave frequency bands corresponds to the linear physical system polyharmonic excited.

Figure 2 illustrates an acceleration spectrum represented in frequency bands centred as follows: f, 2f, 3f, 4f, …, nf and containing sub harmonic signals 1/2f and over harmonic signals 3/2f, 5/2f, 7/2f, ... One could conclude that a part of the sub harmonic and over harmonic signals lead to the exceeding of the permissible level as compared with the basic signal case when for the linear system all the values have been in the frame of the given limit.

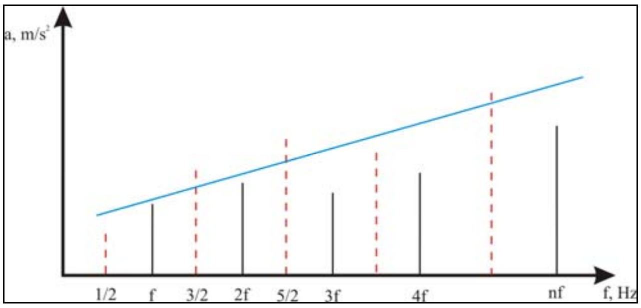

Figure 2. Acceleration spectrum

4. CONCLUSIONS

In the frame of the human body-machine relation, in case of a linear system with external polyharmonic excitation represented by force or acceleration, the system response signal can be presented in j-multiple frequency bands (j = 1, 2,…) and can be compared with the permissible level stated as a function of vibration exposure time.

In situation the system is nonlinear (elastic, dissipating) or during the operation, its behaviour is significant nonlinear, the initial basic signal is

”enriched” by spectrum completion with $i\omega$ and $k\omega$-order harmonics, where $2i = 2j - 1$ and $2k = 2j + 1$, so that this aspect has to be taken into account while comparing with the permissible values.

Moreover, in case of physical systems where during the working process the vibrations are transmitted to human body, it is necessary to evaluate the possibility of some sub harmonic and over harmonic signals appearance that can eliminate the accepted normal operation from the permissible level.

Aiming to correctly evaluate the human protection degree, the vibration monitoring while operating the machine is necessary until technical resource is finished.

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