Some Aspects Regarding the Transition from Beam to Plate Behavior of Vibrating Structures

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Abstract: - The paper introduces a new vision about the vibrational behavior of beams and plates, aiming to show the correlation between vibration modes which nowadays are considered of different types. An analysis was performed by means of the finite element method (FEM) on a structure with constant length and thickness, for which the width was progressively increased. The evolution of resulted natural frequencies and mode shapes in respect to the structure width was observed and described by diagrams and illustrations. In this way it was possible to make a link between certain vibrations characteristic for beams with that reflecting the vibration of plates.

Keywords: - Euler-Bernoulli beam, rectangular plate, vibration, frequency, mode shape, FEM.

1. INTRODUCTION

Plate theory is very like the beam theory, the difference consisting only in an extra dimension [1]. Both the plate and beam theories are referred to as classical strength of materials theories. For beams the next assumptions are made: a straight line perpendicular to the neutral axis of the plate or beam is inextensible, remains straight and only rotates about the undeformed axis [2]. In classical plate theory these assumptions are extended to thin planar bodies, in which the geometry is now slender in only one direction. This result in a two-dimensional problem where deformations are functions of two in-plane coordinates [2]. In beam theory bending and extension is considered in only one direction, since in plate theory these are considered (usually) in two orthogonal directions. In addition, the plate not only bends, but torsion may also occur. Therefore, the vibration analysis of plates is more complicated than that of beams.

In damage detection it is only necessary to know the strain energy distribution. This can be deduced from the curvature (or bending moment) of the elastic beam or plate. Since the problem of beams and plates is in some way similar [3], and plates are more difficult to be solved, the idea is to approach the plates with the beam theory. In early researches regarding vibration of plates, Jacob Bernoulli treated these as a grid of beams subjected only to bending. Sophie Germain succeeds finding the correct formulation for the differential equation for this case [4]. In our previous researches we proved that the mode shapes of a rectangular plate can be composed from the orthogonal bending mode shapes of beams [5]-[7]. This paper investigates the boundaries between where the behavior of a structure cross from beam in plate mode.

2. VIBRATION OF SLENDER BEAMS

The bending vibration of prismatic beams, Fig. 1, is described by the well-known mathematical relation:

$$EI\frac{\partial^4 y}{\partial x^4}+\rho A\frac{\partial^2 y}{\partial t^2}=0 \qquad (1)$$

where the parameters E, I, A and ρ are respectively the Young Modulus, moment of inertia, cross section area and mass density of the analyzed beam of length L.

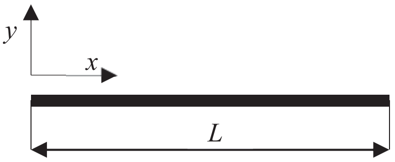


Figure 1. Prismatic beam with length L

The solution of equation (1) can be written as:

$$y(x,t)=w(x)u(t) \qquad (2)$$

Relation (2) separates the spatial and temporal component. Thus, the following characteristic equation that relates the angular frequency ω to the wave number λ, is:

$$\omega^2=\frac{EI}{\rho A}\lambda^4 \qquad (3)$$

The spatial component can be written as:

$$\begin{aligned}w(x)&=C_1\sin(\lambda x)+C_2\cos(\lambda x)+\\&+C_3\sinh(\lambda x)+C_4\cosh(\lambda x)\end{aligned} \qquad (4)$$

Because the forces and moments at the free-free beam's ends are null, the boundary conditions for this support type are:

$$\begin{cases}w''(0)=0\\w'''(0)=0\\w''(L)=0\\w'''(L)=0\end{cases} \qquad (5)$$

and lead to the system of algebraic equations:

$$\begin{cases}-C_2+C_4=0\\-C_1+C_3=0\\-C_1\sin(\lambda L)-C_2\cos(\lambda L)+C_3\sinh(\lambda L)+\\\quad+C_4\cosh(\lambda L)=0\\-C_1\cos(\lambda L)+C_2\sin(\lambda L)+C_3\cosh(\lambda L)+\\\quad+C_4\sinh(\lambda L)=0\end{cases} \qquad (6)$$

A non trivial solutions is obtained if the determinant,

$$\begin{bmatrix}\sinh(kL)-\sin(kL) & \cosh(kL)-\cos(kL)\\\cosh(kL)-\cos(kL) & \sinh(kL)-\sin(kL)\end{bmatrix}$$

resulted from the two last equations in (6) is null. In this case, the system is expressed in a condensed form, that is:

$$\cosh(\lambda L)\cos(\lambda L)=1 \qquad (7)$$

The transcendental equation (7) has an infinite number of solutions. It can be solved numerically, and lead to values from which the first six are presented in the table below.

Table 1. The first five solutions

Mode number i	$\lambda_i L$
0	0
1	4.73008
2	7.85321
3	10.99564
4	14.13715
5	17.27879

Introducing these values back in equation (3) gives the natural frequencies ω_i. The mode shapes are given by the following expression:

$$\begin{aligned}w_i(x)&=[\cos(\lambda_i x)+\cosh(\lambda_i x)]-\\&-\frac{\cos(\lambda_i L)-\cosh(\lambda_i L)}{\sin(\lambda_i L)-\sinh(\lambda_i L)}[\sin(\lambda_i x)+\sinh(\lambda_i x)]\end{aligned} \qquad (8)$$

In figure 2 the first six mode shapes for a free-free beam are presented.

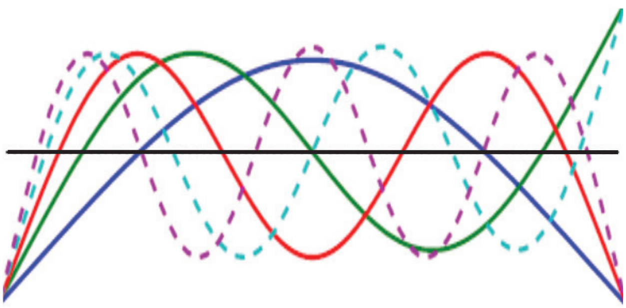


Figure 2. First six mode shapes for a free-free beam

Having a look in table 1 and analyzing figure 2 one can remark that a mode shape with $w = 0$ exists. This indicates that two rigid body modes occur in the xOy plane, one translation and one rotation.

For a beam clamped at both ends, the boundary conditions, motivated by vanishing displacement and slope at the fixed ends, is stated below.

$$\begin{cases} w(0)=0 \\ w'(0)=0 \\ w(L)=0 \\ w'(L)=0 \end{cases} \tag{9}$$

It is interesting to note that the resulted system of algebraic equations is identical with that achieved for the beam with two free ends. As a consequence, the resonance frequencies are the same as in the case of a free-free beam, except that in this case we do not have the two rigid body modes since it is not allowed by the boundary conditions [2].

The mode shapes for the double-clamped beam can be written:

$$w_i(x) = -[\cos(\lambda_i x) - \cosh(\lambda_i x)] + \\ + \frac{\cos(\lambda_i L) - \cosh(\lambda_i L)}{\sin(\lambda_i L) - \sinh(\lambda_i L)}[\sin(\lambda_i x) - \sinh(\lambda_i x)] \tag{10}$$

The idea presented in this paper is that by combining these modes shapes one can attain any mode shape of a rectangular plate clamped on two parallel edges, but also of the double-clamped beam.

3. TYPICAL RECTANGULAR PLATE MODE SHAPES

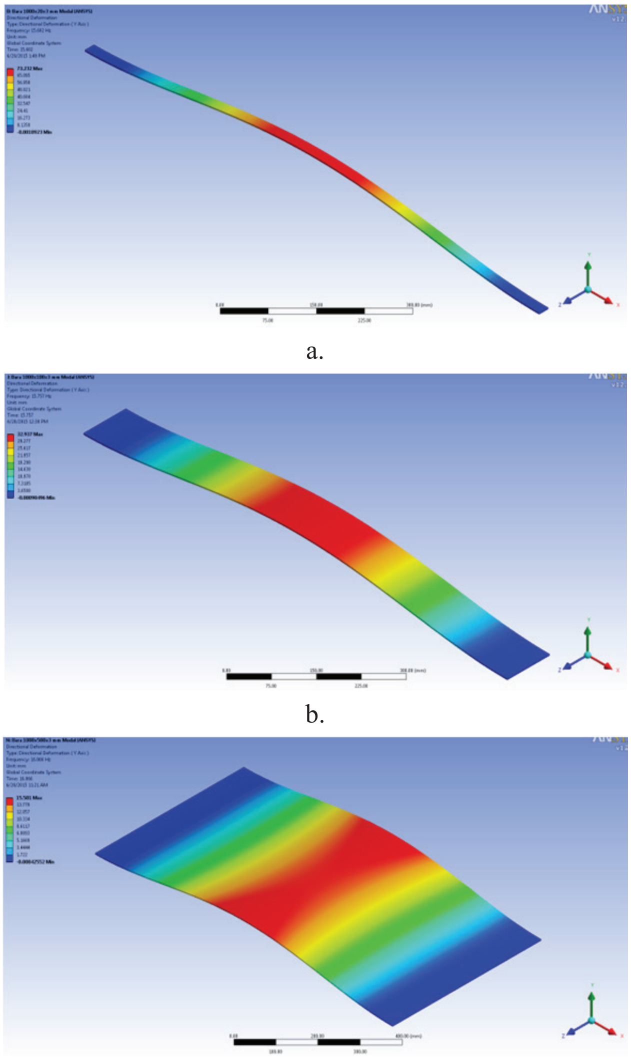
In this section typical mode shapes, attained by analysis performed by the finite element method, are presented. The analyzed specimens are steel bodies, with the length L = 1000 mm (in x direction), thickness h = 3 mm (in y direction) and several dimensions for the width, starting from w_1 = 10 mm and finishing with w_{14} = 500 mm (in z direction). Between 10 mm and 100 mm the width was increase with a step of 10 mm, afterwards with a step of 100 mm until the width of 500 mm is achieved.

For all cases the material is *structural steel*, for which the mechanical and physical characteristics presented in table 2 are associated in the ANSYS library.

Table 2. Mechanical properties of the steel specimens

Yield strength	Elastic modulus	Poisson's ratio	Mass density
[N/mm^2]	[N/mm^2]	[-]	[kg/mm^3]
530	$2\cdot10^5$	0.3	$7.850\cdot10^{-9}$

As a first mode shape type, the weak-axis bending is presented. Figure 3 shows the deflections for the weak-axis bending mode shape one for structures with the width of 20 mm, 100 mm and 500 mm, respectively. One can remark the similitude between the three pictures, demonstrating that this mode shape is common for beams and plates.



a.

b.

c.

Figure 3. Weak-axis bending mode shape one for structures with the width of 20 mm, 100 mm and 500 mm.

Figure 4 present the natural frequency evolution for this mode for different values of the structure width. One can remark that, by increasing the structure's width w, the values of frequency converge to a given level. However, the increase is less then 3.2%.

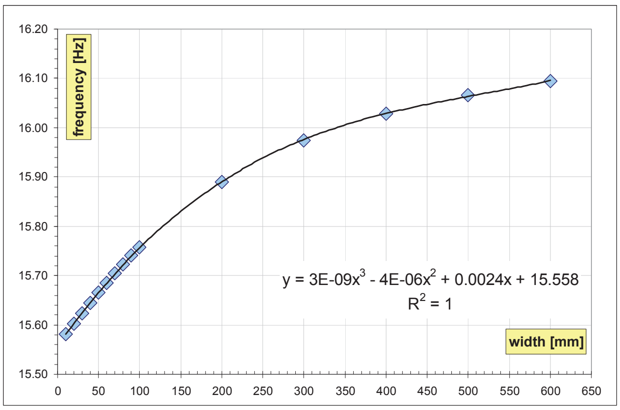
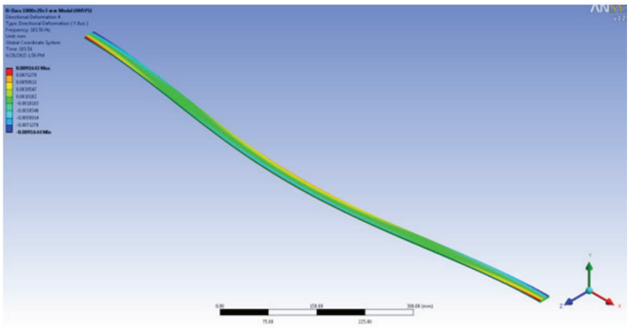


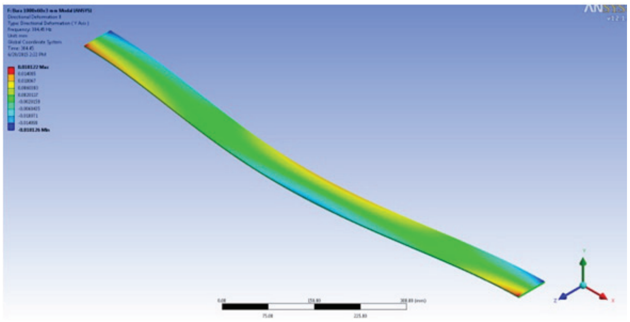
Figure 4. Frequency evolution for the first weak-axis bending mode by increasing the structure's width.

Similar findings were made for the superior modes of the weak-axis bending vibration. Obviously, points located at the same distance x taken from one end achieve the same value of y. This leads us to the conclusion that the global shape can be composed from a bending mode shape of a clamped-clamped beam, as described in equation (10) and a mode shape of a free-free beam described by the equation (8) (just translation, w=0). Both displacements are in the vertical plane – y direction.

A second case consists in the strong-axis bending vibration, where the shape for the vibration mode one is represented in figure 5. Similar with the previous case, the global shape can be composed from a bending mode shape of a clamped-clamped beam and a translational mode shape of a free-free beam (w=0), both in the z direction.



a.



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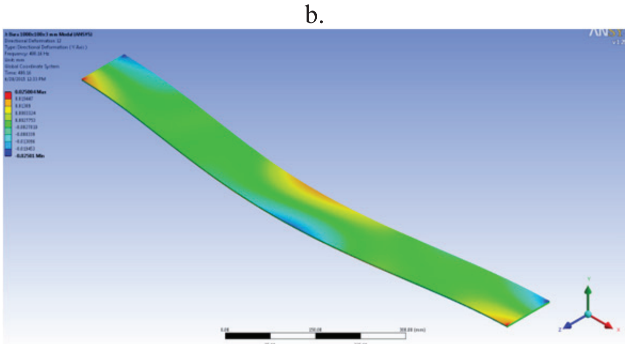


c.

Figure 5. Strong-axis bending mode shape one for structures with the width of 20 mm, 60 mm and 100 mm.

Again similar findings were made for the superior modes of the strong-axis bending vibration, but note that their frequency rapidly increase and so these modes vanish from the first 20 attained modes.

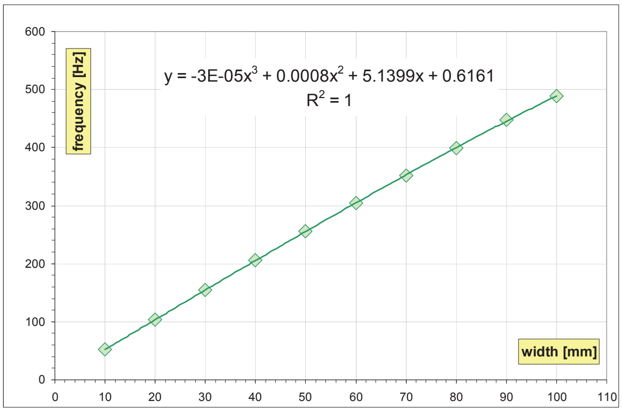
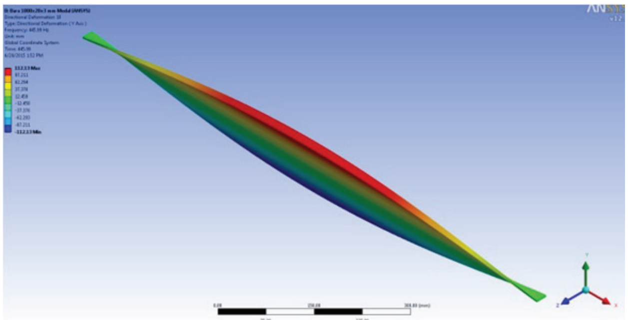


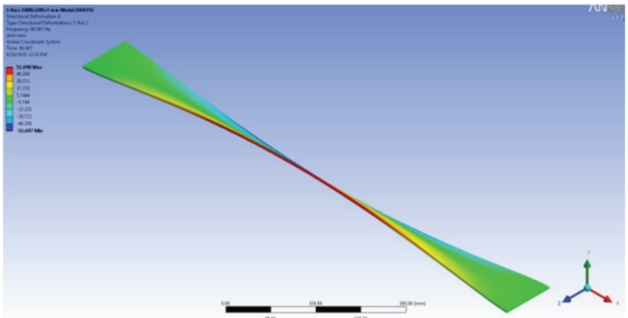
Figure 6. Frequency evolution for the first strong-axis bending mode by increasing the structure's width.

The rapid increasing of the natural frequencies of the strong-axis bending vibration modes make these modes "invisible" for plates, being thus associated just with the vibration of beams.

The third case in our report presents the extension of the torsional beam vibrations to plate-like structures.



a.



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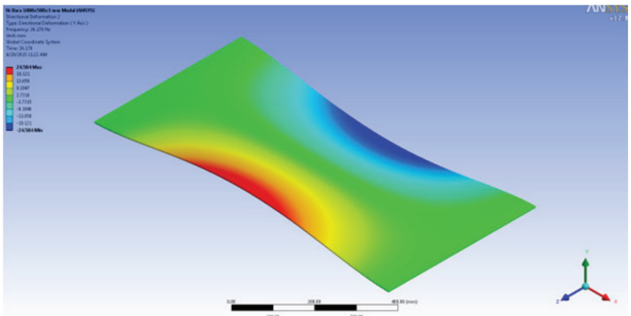
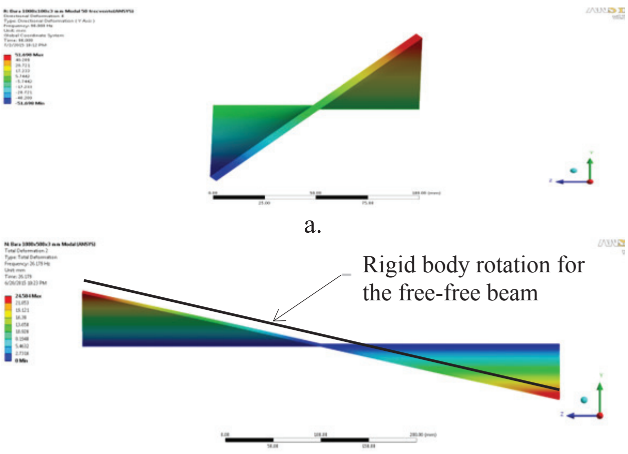


c.

Figure 7. "Torsional" mode shape one for structures with the width of 20 mm, 100 mm and 500 mm.

Figure 7 present the shape attained by the structures by maximum deformation in the mode one for this case. One can remark the similarity for all structure widths. The cross-section shape in plane *yOz* is unchanged at any position *x,* only a rotation being visible. An example at the mid-span, where the most important deformation is achieved, is presented in figure 8: the first picture shows the cross-section of a structure with the width of 100 mm while the second present a much large structure (*w* = 500 mm). This proves that the cross-section in the *yOz* plane (which is in fact rectangular) behaves like a very slim beam that is subjected to mode shape one as free-free beam (rigid body rotation).



a.

b.

Figure 8. Mid-span cross-section in plane *yOz* trough a plate in the first "torsional" vibration mode: a) plate width 100 mm; b) plate width 500 mm.

Moreover, in figure 9 the frontal view of the plate is presented. It is obvious that the longitudinal shape is similar to that of a double-clamped beam in bending mode one, illustrated in the same figure with black line.

This leads us to the conclusion that the "torsional" mode shape of the herein analyzed structure can be composed from orthogonally slim beams deformed in the two ways: bending and free-free (rigid rotation).

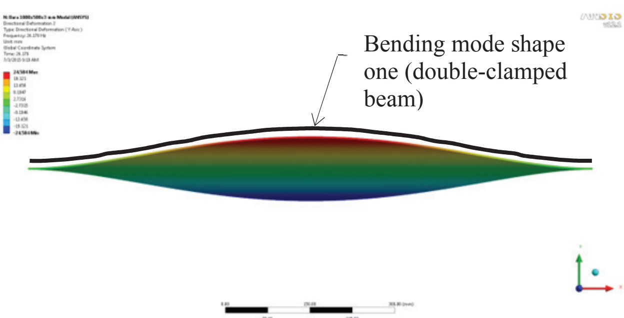


Figure 9. Frontal view of a plate in the first "torsional" vibration mode (*w* = 500 mm).

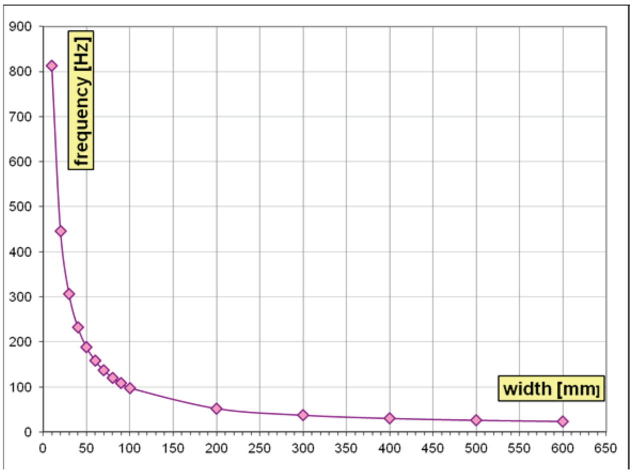
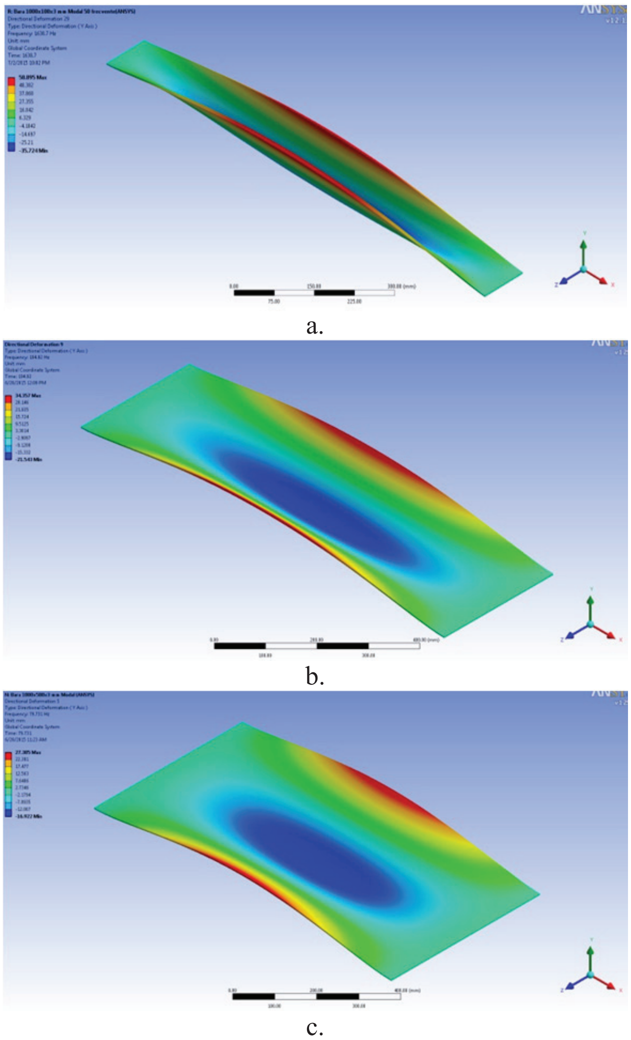


Figure 10. Frequency evolution for the first "torsional" mode by increasing the structure's width.

Regarding the frequency of the first "torsional" mode, by extending the structure in the *z* direction it drastically decrease until a given width *w* is achieved, as shown in figure 10. Afterwards, the width increase produces a slow frequency decrease.

The last identified case is presented in figure 11.



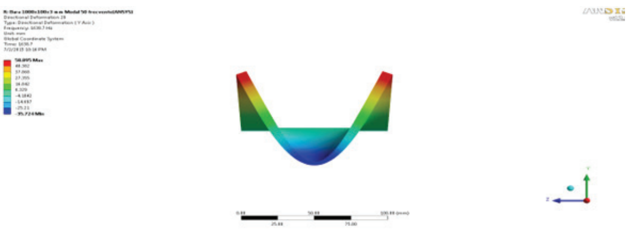
a.

b.

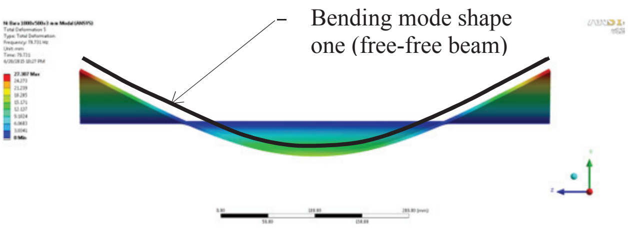
c.

Figure 11. Transversal mode shape one for structures with the width of 100 mm, 300 mm respectively 500 mm.

In this case, nominated as transversal vibration, the structure deforms in the yOz plane as shown in the figure 11 for two plate widths.



a.



b.

Figure 12. Mid-span cross-section in plane yOz trough a plate for the first transversal vibration mode: a) plate width 100 mm; b) plate width 500 mm.

The frontal view, figure 13, proves again that the shape (upper border) is that of a bending beam.

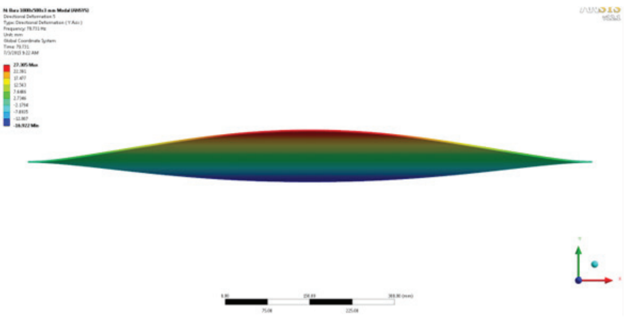


Figure 13. Frontal view of a plate in the first transversal vibration mode (w = 500 mm).

Regarding the natural frequencies, these are not present in the first twenty ones achieved by measurements for the structures with a reduced width (w < 100 mm). An increase of the structure's width determines a frequency decrease; it is severe in the low width domain (typical for beams) and becomes mild for larger structures (typical for plates). This can be observed in figure 14.

By examination of the cases one, three and four it is obvious that these belong to a single class, where the neutral surface in the horizontal plane xOz deforms. In all cases, in the x direction the mode shape of a double-clamped beam is visible, since in the z direction the shape of a free-free beam can be remarked. For the free-free vibration, the first case (figure 3) is a rigid body translation, for the third case (figure 7) the rigid body rotation is evident while for the fourth case (figure 11) a lateral bending is observed. As a result, just one vibration manner is evident for the displacement of the neutral plane xOz in the y direction.

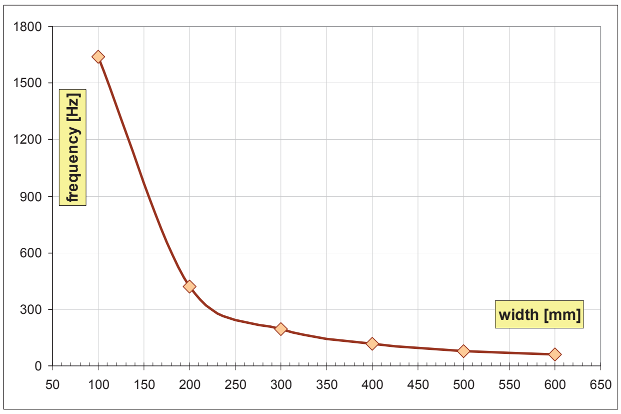


Figure 14. Frequency evolution for the first transversal mode by increasing the structure's width.

In a similar way can be described the vibration of the neutral surface in the xOy plane. In this case, due to dramatically increased stiffness, only few modes are identifiable among the first 20 vibration modes achieved by simulation.

4. CONCLUSION

The paper investigates the dynamics of vibrating structures in order to establish borders between its behavior as beam or plate. For the analyzed structure it was found that the frequencies of the "torsional" and "transversal" modes are relevant to characterize the behavior change. For a width w < 50 mm drastic frequency drop occurs and beam characteristics are present, since for widths w > 100 mm the frequency decrease slowly and a behavior as plate is remarked. This should indicate that the border can be presumed at a value between 50 and 100 mm. Future researches will be designed to highlight the influence of the thickness upon this border.

A relevant aspect arisen from this study is the possibility to derive the structure's shape from the two orthogonal mode shapes of beams with properly set boundary conditions. This results in the fact that the dynamics of beams and plates can be identically described. The false opinion that the two types of structures behave in different way is given by the 'visibility" of specific vibration modes among the modes that are identified by simulations or measurements. The visibility is defined by the value of the frequencies; in most research just the first few modes (12 to 20) are considered, and seldom analysis is made in function of the mode type.

As a consequence, both the “torsional” and “transversal” modes are belong to the same class and can be expressed as a combination of bending modes. This is not so evident for structures with reduced width, where the transversal cross-section is close to a square, but can be easily observed for larger structures.

Based on this finding, a new approach in interpreting the vibration modes is introduced by the authors. It presents clear advantages in the vibration analysis procedures due to the high level of generalization.

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