Vibration of Square Plate with Parabolic Temperature Variation

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Abstract: - Here the effect of parabolic temperature variation (along both the axes) on the natural frequency of non-uniform and non-homogeneous square plate is presented. For non uniformity and non homogeneity, circular variation in thickness and Poisson's ratio is viewed in one direction. Rayleigh Ritz technique has been applied to solve the frequency equation. The results (first two modes) of the study are presented with the help of graphs. A comparison is also made with the existing result.

Keywords: - Vibration, Square plate, Parabolic, Temperature variation.

NOMENCLATURE

a Length of the plate
x, y Coordinates in the plane of plate
M_x, M_y Bending moment intensities in x and y direction
M_{xy} Twisting moment intensity
Y Young's modulus
ν Poisson's ratio
D_1 Flexural rigidity
ρ Mass density per unit volume of the plate material
t Time
$\phi(x, y, t)$ Deflection of plate
$\Phi(x, y)$ Deflection function
$T(t)$ Time function
l Thickness of plate at x, y
m Non-homogeneity in the plate's material
β Tapering parameter
α Temperature gradient
λ Frequency parameter

1. INTRODUCTION

The behavior (dynamic) of plate structures is significant to many applications in sciences and engineering. Moreover, non-uniform and non-homogeneous plates are integral parts of power plants, aerospace industry, machines, ship building, nuclear, ocean and naval engineering. The vibration characteristics of non-uniform and non-homogeneous plate coupled with temperature is rigorously treated and well documented in relevant literature. A few published works on plates are available in this direction.

Vibrational analysis of plates based upon classical plate theory (CPT) has been provided by Leissa [1-5]. He analyzed vibration of different structure (plates of different shape) on different boundary (clamped, simply supported and free) conditions.

An asymmetric vibration of circular plates (non-homogeneous) analyzed by Sharma et al. [6] on the basis of classical plate theory. They considered parabolically varying thickness in the plate and exponential variation in Young's modulus as well as density along radial direction as a non-homogeneity effect. They showed the effects of taper parameter, non-homogeneity parameter, density parameter and nodal diameter on vibration modes using Ritz method.

Zhang et al. [7] presented an improved Fourier series method to analyze free vibration of the rectangular plate with non-uniform boundary conditions. They treated all the coefficients of expansion series as the generalized coordinates and determined them using the Rayleigh Ritz technique. They also claimed that the presented method can universally applied to a wide spectrum of plate vibration problems. They compared the obtained result with FEM to show validity of their results.

Sharma et al. [8] studied the natural vibration of orthotropic rectangle plate. They studied parabolic variation in temperature and thickness of the plate. They calculated time period, deflection function and logarithmic decrement for first two modes of vibrations at different values of plate's parameters. Sharma and Sharma [9] studied natural frequency of non-uniform and non-homogeneous parallelogram plate. For non-uniformity and non-homogeneity, they considered linear variation in thickness and density. Sharma et al. [10] presented Rayleigh Ritz method to analyze vibration of C-S-C-S trapezoidal plate. In this paper they considered variation in thickness (parabolic in one and linear variation in other), density (linear in one direction) and temperature variation (linear in both direction). They provided first two modes of vibration corresponding to thermal gradient, taper constant, aspect ratio and non-homogeneity parameters.

Chen et al. [11] investigated transverse vibration of visco elastic Timoshenko beam columns by using generalized Hamilton principle. They established orthogonality conditions in the state space. A numerical illustration has been given in this paper which shows the effect of length to depth ratio, the axial tension, and the viscosity coefficients on the natural frequencies and the decrement coefficients.

Variational iteration method (VIM) is developed by Torabi et al. [12] for free vibration analysis of a Timoshenko beam with different boundary conditions. In this, they chosen an appropriate Lagrange multiplier (depending on order of the differential equation) for the problem after that an iteration process is used till the desired accuracy is achieved. They showed that the accuracy of VIM is approximately same as the exact solution and much better than the differential quadrature method (DQM) for solving the free vibration of a Timoshenko beam.

Sharma et al. [13, 14] provided free vibration of square plate. They studied the effect of non-uniformity (exponential and circular variation in thickness) and non-homogeneity effect (circular variation in density and Poisson's ratio) along with temperature effect (linear and parabolic) on vibrational frequencies.

An exact solution for free transverse vibrations of a Timoshenko beam carrying multiple arbitrary concentrated masses investigated by Torabi et al. [15] with various boundary conditions. They examined the effects of value, position and number of concentrated masses on acquired mode shapes for different boundary conditions.

Soni et al. [16] proposed an analytical model for vibration analysis of partially cracked rectangular (isotropic) plates coupled with fluid medium. Based on CPT, the governing equation of motion of the plate has been modified. They evaluated fundamental frequencies by expressing the lateral deflection in terms of modal functions. In this study they presented new results for fundamental frequencies affected by crack length, fluid level, fluid density and immersed depth of plate.

An analytical model for nonlinear vibrations in rectangular (cracked and isotropic) plate has been presented by Joshi et al. [17]. They also modified the equation of motion based on CPT to accommodate the effect of internal cracks using the Line Spring Model. The natural frequencies of the cracked plate have been calculated by using Galerkin's method with three different boundary conditions They also showed that crack parallel to the longer side of the plate affect the vibration characteristics more as compared to crack parallel to the shorter side. The analytical results obtained are also assessed with FEM results.

Here author studies the effect of bi-parabolic temperature variation on clamped square plate with circular variation in thickness and Poisson's ratio. All the results are presented with the help of figures. A graphical comparison is also given to support the results of present paper.

2. ANALYSIS OF MOTION

For transverse motion the differential equation is

$$\frac{\partial^2 M_x}{\partial x^2}+2\frac{\partial^2 M_{xy}}{\partial x\partial y}+\frac{\partial^2 M_y}{\partial y^2}=\rho l\frac{\partial^2 \phi}{\partial t^2} \tag{1}$$

where

$$\left.\begin{aligned} M_x &= -D_1\left(\frac{\partial^2\phi}{\partial x^2}+\nu\frac{\partial^2\phi}{\partial y^2}\right)\\ M_y &= -D_1\left(\frac{\partial^2\phi}{\partial y^2}+\nu\frac{\partial^2\phi}{\partial x^2}\right)\\ M_{xy} &= -D_1\left(1-\nu\right)\frac{\partial^2\phi}{\partial x\partial y}\end{aligned}\right\} \tag{2}$$

Using Eq. (2), Eq. (1) becomes

$$\left[\begin{aligned} &D_1\left(\frac{\partial^4\phi}{\partial x^4}+2\frac{\partial^4\phi}{\partial x^2\partial y^2}+\frac{\partial^4\phi}{\partial y^4}+\frac{\partial^2\nu}{\partial x^2}\frac{\partial^2\phi}{\partial y^2}\right)\\ &+2\frac{\partial D_1}{\partial x}\left(\frac{\partial^3\phi}{\partial x^3}+\frac{\partial^3\phi}{\partial x\partial y^2}+\frac{\partial\nu}{\partial x}\frac{\partial^2\phi}{\partial y^2}\right)\\ &+2\frac{\partial D_1}{\partial y}\left(\frac{\partial^3\phi}{\partial y^3}+\frac{\partial^3\phi}{\partial y\partial x^2}-\frac{\partial\nu}{\partial x}\frac{\partial^2\phi}{\partial x\partial y}\right)\\ &+\frac{\partial^2 D_1}{\partial x^2}\left(\frac{\partial^2\phi}{\partial x^2}+\nu\frac{\partial^2\phi}{\partial y^2}\right)\\ &+\frac{\partial^2 D_1}{\partial y^2}\left(\frac{\partial^2\phi}{\partial y^2}+\nu\frac{\partial^2\phi}{\partial x^2}\right)\\ &+2\left(1-\nu\right)\frac{\partial^2 D_1}{\partial x\partial y}\frac{\partial^2\phi}{\partial x\partial y}\end{aligned}\right]+\rho l\frac{\partial^2\phi}{\partial t^2}=0 \tag{3}$$

The deflection of the plate can be written as product of deflection function and time function as

$$\Phi\left(x,y,t\right)=\phi\left(x,y\right)*T(t) \tag{4}$$

Substitute Eq. (4) in Eq. (3), we get

$$T\left[\begin{array}{l} D_1\left(\frac{\partial^4\Phi}{\partial x^4}+2\frac{\partial^4\Phi}{\partial x^2\partial y^2}+\frac{\partial^4\Phi}{\partial y^4}+\frac{\partial^2\nu}{\partial x^2}\frac{\partial^2\Phi}{\partial y^2}\right) \\ +2\frac{\partial D_1}{\partial x}\left(\frac{\partial^3\Phi}{\partial x^3}+\frac{\partial^3\Phi}{\partial x\partial y^2}+\frac{\partial\nu}{\partial x}\frac{\partial^2\Phi}{\partial y^2}\right) \\ +2\frac{\partial D_1}{\partial y}\left(\frac{\partial^3\Phi}{\partial y^3}+\frac{\partial^3\Phi}{\partial y\partial x^2}-\frac{\partial\nu}{\partial x}\frac{\partial^2\Phi}{\partial x\partial y}\right) \\ +\frac{\partial^2 D_1}{\partial x^2}\left(\frac{\partial^2\Phi}{\partial x^2}+\nu\frac{\partial^2\Phi}{\partial y^2}\right) \\ +\frac{\partial^2 D_1}{\partial y^2}\left(\frac{\partial^2\Phi}{\partial y^2}+\nu\frac{\partial^2\Phi}{\partial x^2}\right) \\ +2(1-\nu)\frac{\partial^2 D_1}{\partial x\partial y}\frac{\partial^2\Phi}{\partial x\partial y} \end{array}\right]+\rho l\Phi\frac{\partial^2\Phi}{\partial t^2}=0 \quad (5)$$

Now separate the variable using variable separable technique, we get

$$\frac{\left[\begin{array}{l} D_1\left(\frac{\partial^4\Phi}{\partial x^4}+2\frac{\partial^4\Phi}{\partial x^2\partial y^2}+\frac{\partial^4\Phi}{\partial y^4}+\frac{\partial^2\nu}{\partial x^2}\frac{\partial^2\Phi}{\partial y^2}\right) \\ +2\frac{\partial D_1}{\partial x}\left(\frac{\partial^3\Phi}{\partial x^3}+\frac{\partial^3\Phi}{\partial x\partial y^2}+\frac{\partial\nu}{\partial x}\frac{\partial^2\Phi}{\partial y^2}\right) \\ +2\frac{\partial D_1}{\partial y}\left(\frac{\partial^3\Phi}{\partial y^3}+\frac{\partial^3\Phi}{\partial y\partial x^2}-\frac{\partial\nu}{\partial x}\frac{\partial^2\Phi}{\partial x\partial y}\right) \\ +\frac{\partial^2 D_1}{\partial x^2}\left(\frac{\partial^2\Phi}{\partial x^2}+\nu\frac{\partial^2\Phi}{\partial y^2}\right) \\ +\frac{\partial^2 D_1}{\partial y^2}\left(\frac{\partial^2\Phi}{\partial y^2}+\nu\frac{\partial^2\Phi}{\partial x^2}\right) \\ +2(1-\nu)\frac{\partial^2 D_1}{\partial x\partial y}\frac{\partial^2\Phi}{\partial x\partial y} \end{array}\right]}{\rho l\Phi}=-\frac{1}{T}\frac{\partial^2\Phi}{\partial t^2}=\omega^2\,(\text{say}) \quad (6)$$

Taking first and last term of Eq. (6), we get

$$\left[\begin{array}{l} D_1\left(\frac{\partial^4\Phi}{\partial x^4}+2\frac{\partial^4\Phi}{\partial x^2\partial y^2}+\frac{\partial^4\Phi}{\partial y^4}+\frac{\partial^2\nu}{\partial x^2}\frac{\partial^2\Phi}{\partial y^2}\right) \\ +2\frac{\partial D_1}{\partial x}\left(\frac{\partial^3\Phi}{\partial x^3}+\frac{\partial^3\Phi}{\partial x\partial y^2}+\frac{\partial\nu}{\partial x}\frac{\partial^2\Phi}{\partial y^2}\right) \\ +2\frac{\partial D_1}{\partial y}\left(\frac{\partial^3\Phi}{\partial y^3}+\frac{\partial^3\Phi}{\partial y\partial x^2}-\frac{\partial\nu}{\partial x}\frac{\partial^2\Phi}{\partial x\partial y}\right) \\ +\frac{\partial^2 D_1}{\partial x^2}\left(\frac{\partial^2\Phi}{\partial x^2}+\nu\frac{\partial^2\Phi}{\partial y^2}\right) \\ +\frac{\partial^2 D_1}{\partial y^2}\left(\frac{\partial^2\Phi}{\partial y^2}+\nu\frac{\partial^2\Phi}{\partial x^2}\right) \\ +2(1-\nu)\frac{\partial^2 D_1}{\partial x\partial y}\frac{\partial^2\Phi}{\partial x\partial y} \end{array}\right]-\rho l\omega^2\Phi=0 \quad (7)$$

Eq. (7) represents the transverse motion of the plate with variable D_1 (flexural rigidity of the plate) and ν (Poisson's ratio). The expression for flexural rigidity of the plate is $D_1=\frac{Yl^3}{12\left(1-\nu^2\right)}$

3. SOLUTION FOR FREQUENCY EQUATION AND VIBRATIONAL MODES

We are using Rayleigh Ritz technique (i.e., maximum kinetic energy K_s must equal to maximum strain energy V_s) to solve equation of motion, therefore we have

$$\delta\left(K_s-T_s\right)=0 \quad (8)$$

Here the expression for kinetic energy K_s and strain energy V_s is given as

$$K_s=\frac{1}{2}\rho\omega^2\int_0^a\int_0^a l\,\Phi^2\,dydx \quad (9)$$

$$V_s=\frac{1}{2}\int_0^a\int_0^a D_1\left\{\begin{array}{l}\left(\frac{\partial^2\Phi}{\partial x^2}\right)^2+\left(\frac{\partial^2\Phi}{\partial y^2}\right)^2 \\ +2\nu\frac{\partial^2\Phi}{\partial x^2}\frac{\partial^2\Phi}{\partial y^2} \\ +2\left(1-\nu\right)\left(\frac{\partial^2\Phi}{\partial x\partial y}\right)^2\end{array}\right\}dydx \quad (10)$$

3.1. Assumptions Required

As plate's vibration is very vast field of research, therefore the present paper requires some limitations in the form of assumptions.

(a). Since the plate is non-uniform therefore author assume that the thickness of plate varies circularly in one direction as shown in figure 1.

$$l=l_0\left\{1+\beta\left(1-\sqrt{1-\frac{x^2}{a^2}}\right)\right\} \quad (11)$$

Where $\beta,\left(0\le\beta\le1\right)$ is known as tapering parameter of the plate and $l=l_0$ at $x=0$

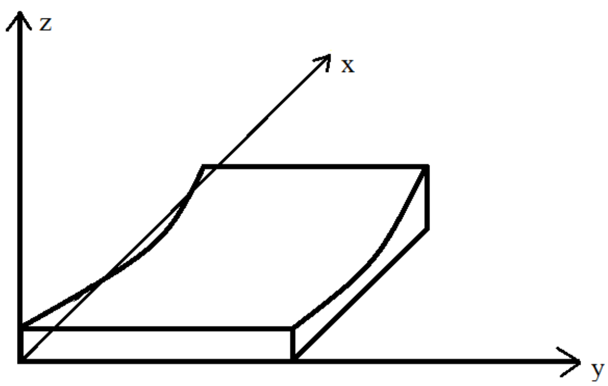

Figure 1. Square plate with one dimensional circular variation in x – direction

(b). For non-homogeneity in the plate's material, author assume that Poisson's ratio of the plate varies circularly in one direction as

$$\nu = \nu_0 \left\{ 1 - m\left(1 - \sqrt{1 - \frac{x^2}{a^2}} \right) \right\} \tag{12}$$

where $m, (0 \le m \le 1)$ is known as non-homogeneity constant.

(c). The temperature variation on the plate is parabolic along the axes as

$$\tau = \tau_0 \left(1 - \frac{x^2}{a^2} \right)\left(1 - \frac{y^2}{a^2} \right) \tag{13}$$

where τ and τ_0 are known as temperature excess the reference temperature at any point on the plate and at origin i.e. $x = y = 0$.

For engineering material, the modulus of elasticity is

$$Y = Y_0 \left(1 - \gamma \tau \right) \tag{14}$$

where Y_0 is the Young's modulus at $\tau = 0$ and γ is known as slope of variation.

Using Eq. (13), Eq. (14) becomes

$$Y = Y_0 \left\{ 1 - \alpha \left(1 - \frac{x^2}{a^2} \right)\left(1 - \frac{y^2}{a^2} \right) \right\} \tag{15}$$

where $\alpha, (0 \le \alpha < 1)$ is called temperature gradient, which is the product of temperature at origin and slope of variation i.e., $\alpha = \gamma \tau_0$.

Using Eqs. (11), (12) and (15), the flexural rigidity of the plate becomes

$$D_1 = \frac{\left[\begin{array}{l} Y_0 l_0^3 \left[1 - \alpha \left\{ 1 - \frac{x^2}{a^2} \right\}\left\{ 1 - \frac{y^2}{a^2} \right\} \right] \\ \left\{ 1 + \beta \left(1 - \sqrt{1 - \frac{x^2}{a^2}} \right) \right\}^3 \end{array} \right]}{12\left(1 - \nu_0^2 \left\{ 1 - m\left(1 - \sqrt{1 - \frac{x^2}{a^2}} \right) \right\}^2 \right)} \tag{16}$$

(d). In the present scenario, we are computing frequencies on C-C-C-C conditions (all the four edges of the plate are clamped). The boundary conditions for the clamped plate are

$$\begin{aligned} \Phi = \frac{\partial \Phi}{\partial x} = 0,\ x = 0,\ a \\ \Phi = \frac{\partial \Phi}{\partial y} = 0,\ y = 0,\ a \end{aligned} \tag{17}$$

The two-term deflection function which satisfy Eq. (17) could be

$$\begin{aligned} \Phi = \left[\left(\frac{x}{a} \right)\left(\frac{y}{a} \right)\left(1 - \frac{x}{a} \right)\left(1 - \frac{y}{a} \right) \right]^2 \\ \left[Q_1 + Q_2 \left(\frac{x}{a} \right)\left(\frac{y}{a} \right)\left(1 - \frac{x}{a} \right)\left(1 - \frac{y}{a} \right) \right] \end{aligned} \tag{18}$$

where Q_1 and Q_2 represents arbitrary constants.

Substitute Eqs. (11), (12) and (16) in Eqs. (9) and (10), we get

$$K_s = \frac{1}{2} \rho \omega^2 l_0 \int_0^a \int_0^a \left[1 + \beta \left(1 - \sqrt{1 - \frac{x^2}{a^2}} \right) \right] \Phi^2 \, dy dx \tag{19}$$

$$V_s = \frac{Y_0 l_0^3}{24} \int_0^a \int_0^a \left[\begin{array}{l} \dfrac{\left[\begin{array}{l} \left[1 - \alpha \left(1 - \frac{x^2}{a^2} \right)\left(1 - \frac{y^2}{a^2} \right) \right] \\ \left\{ 1 + \beta \left(1 - \sqrt{1 - \frac{x^2}{a^2}} \right) \right\}^3 \end{array} \right]}{\left(1 - \nu_0^2 \left\{ 1 - m\left(1 - \sqrt{1 - \frac{x^2}{a^2}} \right) \right\}^2 \right)} \\ \left\{ \begin{array}{l} \left(\frac{\partial^2 \Phi}{\partial x^2} \right)^2 + \left(\frac{\partial^2 \Phi}{\partial y^2} \right)^2 + 2\left(\frac{\partial^2 \Phi}{\partial x^2} \frac{\partial^2 \Phi}{\partial y^2} \right) \\ \nu_0 \left\{ 1 - m\left(1 - \sqrt{1 - \frac{x^2}{a^2}} \right) \right\} \\ +2\left(1 - \nu_0 \left\{ 1 - m\left(1 - \sqrt{1 - \frac{x^2}{a^2}} \right) \right\} \right) \\ \left(\frac{\partial^2 \Phi}{\partial x \partial y} \right)^2 \end{array} \right\} \end{array} \right] dy dx \tag{20}$$

Now converting x and y into non-dimensional variable X and Y as

$$X = \frac{x}{a}, Y = \frac{y}{a} \tag{21}$$

Using Eq (21), Eqs (19) and (20) becomes

$$K_s = \frac{1}{2} \rho \omega^2 l_0 a^2 \int_0^1 \int_0^1 \left[1 + \beta \left(1 - \sqrt{1 - X^2} \right) \right] \Phi^2 \, dY dX \tag{22}$$

$$V_s = \frac{Y_0 l_0^3}{24 a^2} \int_0^1 \int_0^1 \left[\begin{array}{l} \dfrac{\left[\begin{array}{l} \left[1 - \alpha \left(1 - X^2 \right)\left(1 - Y^2 \right) \right] \\ \left\{ 1 + \beta \left(1 - \sqrt{1 - X^2} \right) \right\}^3 \end{array} \right]}{\left(1 - \nu_0^2 \left\{ 1 - m\left(1 - \sqrt{1 - X^2} \right) \right\}^2 \right)} \\ \left\{ \begin{array}{l} \left(\frac{\partial^2 \Phi}{\partial X^2} \right)^2 + \left(\frac{\partial^2 \Phi}{\partial Y^2} \right)^2 + 2\left(\frac{\partial^2 \Phi}{\partial X^2} \frac{\partial^2 \Phi}{\partial Y^2} \right) \\ \nu_0 \left\{ 1 - m\left(1 - \sqrt{1 - X^2} \right) \right\} \\ +2\left(1 - \nu_0 \left\{ 1 - m\left(1 - \sqrt{1 - X^2} \right) \right\} \right) \\ \left(\frac{\partial^2 \Phi}{\partial x \partial y} \right)^2 \end{array} \right\} \end{array} \right] dY dX \tag{23}$$

Using Eqs. (22) and (23), Eq. (8) becomes

$$\delta\left(V_s - \lambda^2 K_s\right) = 0 \tag{24}$$

where $\lambda^2 = \dfrac{12\rho\omega^2 a^4}{Y_0 l_0^2}$ is known as frequency parameter.

Eq. (24) consists of two unknown's constants i.e., Q_1, Q_2 because of substitution of deflection function Φ. This constant can be determined by

$$\frac{\partial\left(V_s - \lambda^2 K_s\right)}{\partial Q_i} = 0,\ i = 1,\ 2 \tag{25}$$

After simplifying Eq. (25), we get homogeneous system of equation as

$$\begin{aligned} e_{11}Q_1 + e_{12}Q_2 &= 0 \\ e_{21}Q_1 + e_{22}Q_2 &= 0 \end{aligned} \tag{26}$$

In order to obtain non-trivial solution (frequency equation), the determinant of the coefficient matrix from Eq. (26) must be zero. Therefore, we have

$$\begin{vmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{vmatrix} = 0 \tag{27}$$

Eq. (27) is quadratic equation from which we get two roots as λ_1 (Ist Mode) & λ_2 (IInd Mode).

4. RESULT DISCUSSION

Natural frequency of vibration (first two modes) is calculated corresponding to taper constant β, non-homogeneity constant m and thermal gradient α is presented with the help of figures.

Frequency modes corresponding to increasing value of taper constant β is presented in figure 2 for the following values.

(i). $\mathrm{m} = \alpha = 0$, (ii). $\mathrm{m} = \alpha = 0.4$, (iii). $\mathrm{m} = \alpha = 0.8$

It is noted that frequency modes increasing with the increasing value of taper constant β for all the three cases. The rate of increment is not much higher due to circular variation in tapering parameter as in case of exponential variation in tapering parameter. The frequency modes decrease when the non-homogeneity m and temperature α variation on the plate increases from 0 to 0.8.

Figure 3 presents the frequency modes corresponding to increasing value of non-homogeneity m in the plate's material for the following values.

(iv). $\beta = \alpha = 0$, (v). $\beta = \alpha = 0.4$, (vi). $\beta = \alpha = 0.8$

From the figure 3, it can easily see that the frequency modes are slight decreasing (almost constant) with the increasing value of non-homogeneity constant m for all the three values due to circular variation in non-homogeneity m. The frequency modes also decrease when the tapering parameter β and temperature α variation on the plate varies from 0 to 0.8.

Figure 4 represents the frequency modes for increasing value of temperature α variation on the plate for the following values.

(vii). $\beta = \mathrm{m} = 0$, (viii). $\beta = \mathrm{m} = 0.4$, (ix). $\beta = \mathrm{m} = 0.8$

From the figure 4, it is interesting to note that frequency modes decrease with the increasing value of thermal gradient α for the all the three values. On the other part frequency modes increase when tapering parameter β and non-homogeneity m in the plate's material varies from 0 to 0.8.

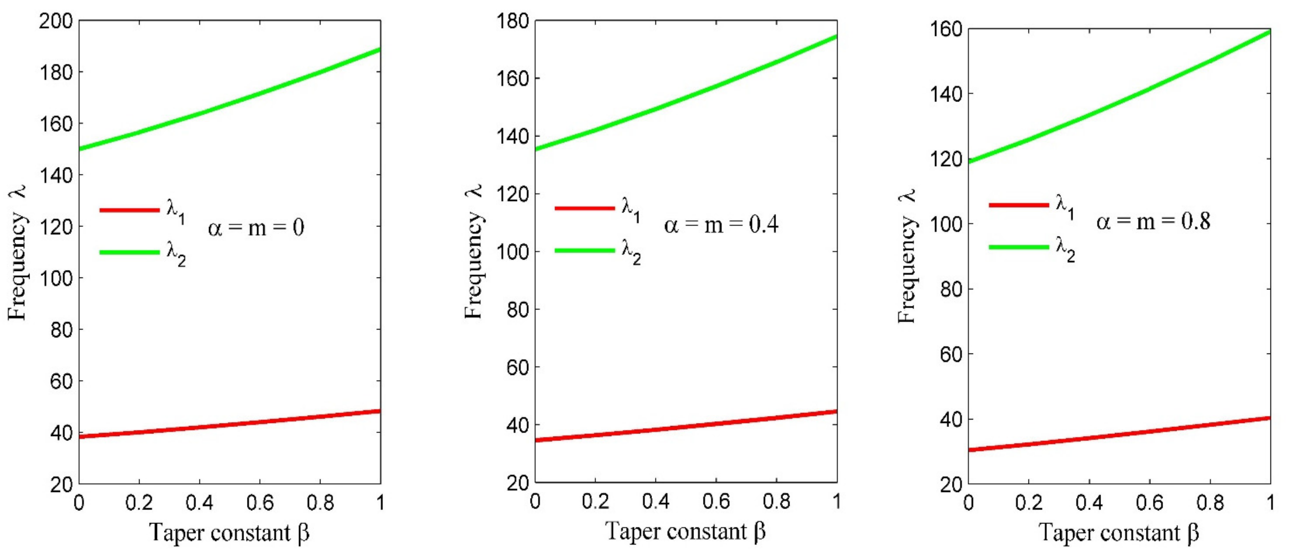

Figure 2. Taper constant β vs frequency

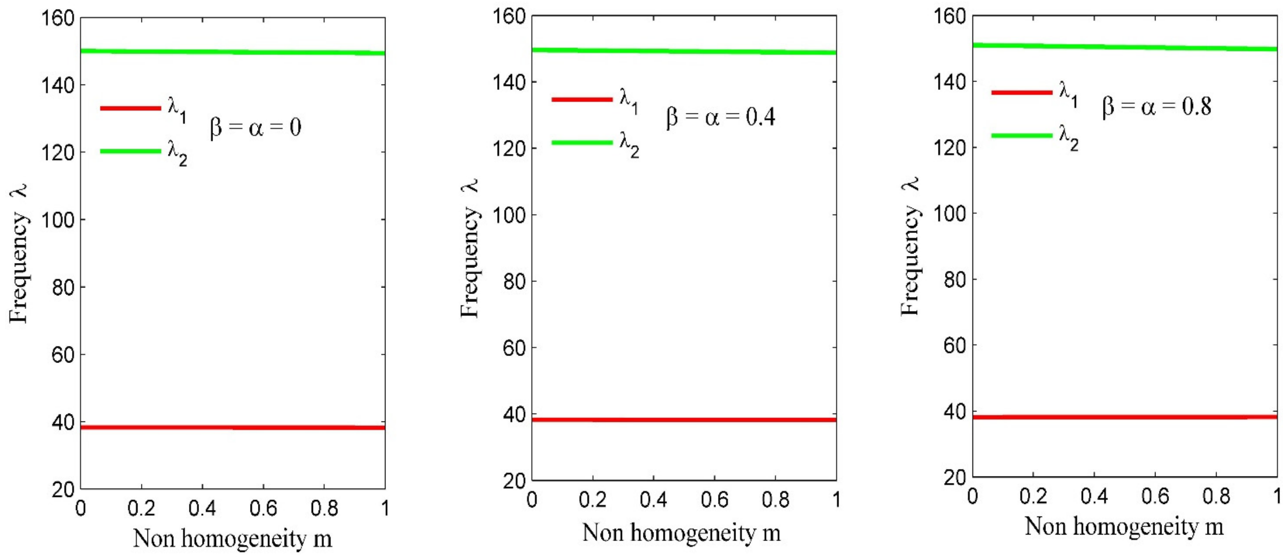

Figure 3. Non homogeneity m vs frequency

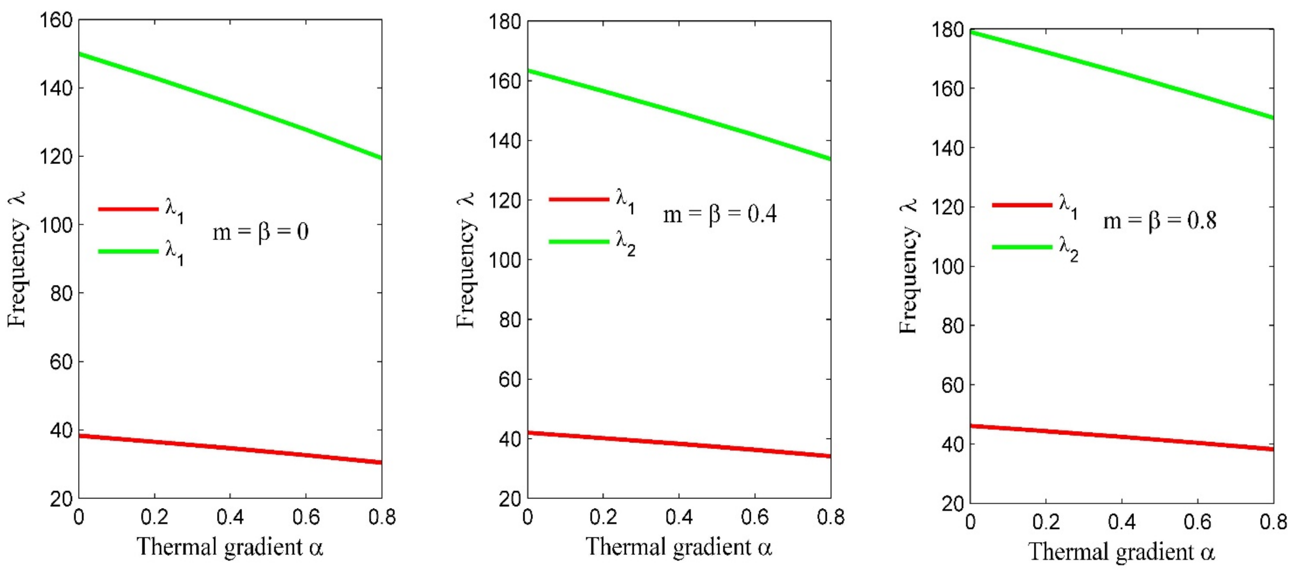

Figure 4. Thermal gradient α vs frequency

5. RESULT COMPARISON

Here, author compare the result of present paper and [14] with the help of figures. Figure 5 show the comparison of frequency modes with [14] corresponding to tapering parameter β for the fixed value of non-homogeneity constant $m = 0.4$ and thermal gradient $\alpha = 0.4$. Figure 5 reflects that frequency modes in present paper are less when compared to [14].

A comparison of frequency modes with [14] corresponding to thermal gradient α is shown in figure 6 for fixed value of tapering parameter $\beta = 0.4$ and non-homogeneity constant $m = 0.4$. From figure 6, we can see that frequency modes in present paper are less when compared to [14]. Frequency modes of present paper and [14] coincide when thermal gradient α on the plate is 0.

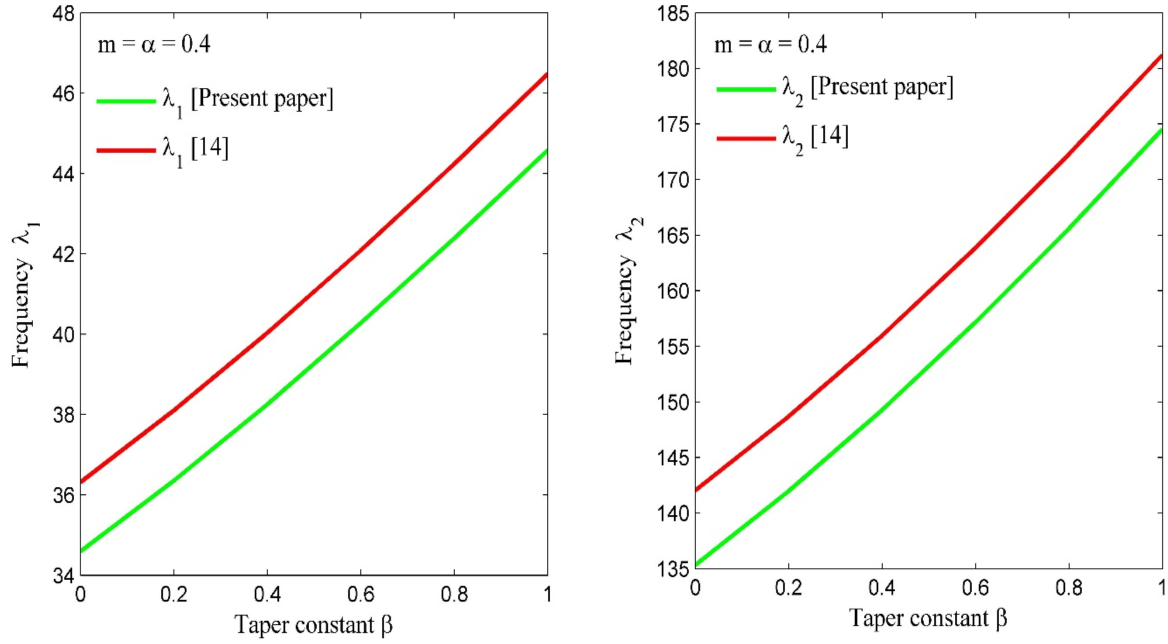

Figure 5. Comparison of frequency modes with [14] corresponding to taper constant β

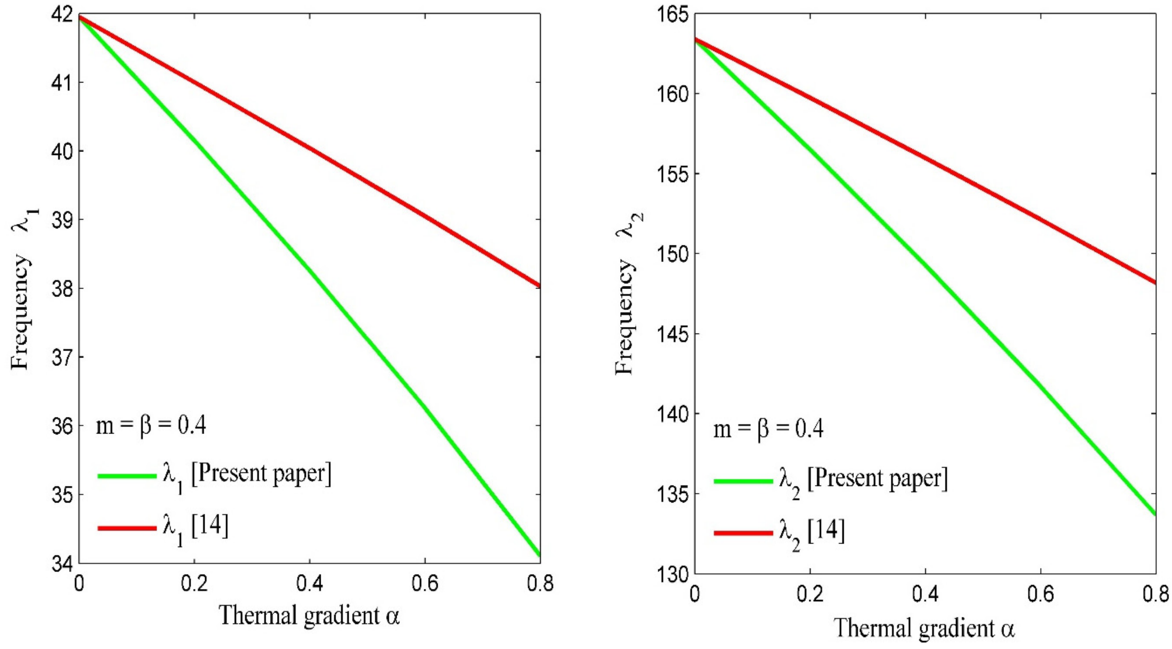

Figure 6. Comparison of frequency modes with [14] corresponding to thermal gradient α

6. CONCLUSION

From the discussion of result and comparison section, author would like to record that frequency modes increase with the increasing value of tapering parameter β. The rate of increment is very less due to circular variation in thickness. Again, due to circular variation in non-homogeneity m, the frequency modes are almost constant in variation. The frequency modes decrease with the increasing value of thermal gradient α.

Due to parabolic temperature variation, the frequency modes are less corresponding to tapering parameter β and thermal gradient α in present paper when compared to [14] (linear variation in temperature) as shown in figure 5 and figure 6.

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